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Ring isomorphism theorems proof pdf

From ProofWiki Let R be a ring, and let it be: S being a subring $$R $$J be ideal R. Then: $(1): \quad S + J$ is a subring $R $ $ (2): \quad J $ is ideal $S + J $ $ (3): \quad S \cap J $ is ideal $S $ $(4): \quad \dfrac S {S \cap J} \cong \dfrac {S + J} J$ where the pool of $\cong$ denotes isomorphism. This decision is also referred to by some sources as the first isomorphism theorem. Evidence of arifiable Relationship can be described by this commative image: $(1): \quad S + J $ is a subring of R Of The Sum of All Ring Products is an Additional Subset, $S + J $ is an additional subset R. Say $s, s' \in S, j, j' \in J$. Then: $\paren {s + j} \paren {s' + j'}$ \(\displaystyle \paren {s + j} \paren {s' + j'}\) \(=\) \\displaystyle s' + b{j' + s ' j + j'}\) \(\displaystyle \) \(\in\) \(\displaystyle S + J\) as $$J is ideal $$R so by Subring Test $S + J $ is a subring R. $\Box $ $(2): \quad J$ is ideal $S + J $ Let $s + j \in S + J $ and let $j \in J$. Then: \(\displaystyle \paren {s + j} j'\) \(=\) \(\displaystyle s j + s j'\) \(\displaystyle \) \(\in\) \(\displaystyle J\) as $s j, s j' \in J$ as $J $ is ideal $R $$J $ is ideal $S + J$. $\Box $ $(3): \quad S \cap J$ is ideal S Let $u: R \to R / J $ being epimorphism picked. Let $u be a $u $$S$ restriction. Then $u': S\to R/J$ is homomorphism. The u' image is a set of all $s + J $ for $s \in S$: $\image {u'} = \dfrac {S + J} J$ Now, isirel u' is a set of all $$S$ elements delivered to $0_{S/J}$ by u. That is, all the elements $S are also in J itself, which is how the quota ring behaves. That is: $\ker \paren {u'} = S \cap J$ and so on from Kernel of Ring Homomorphism is Ideal, $S \cap J$ is ideal S. $(4): \quad \dfrac S {S \cap J} \cong \dfrac {S + J} J$ This follows directly from The First Isomorphism Teorem. $\blacksquare $ Also known as Because of the commative stoning image form, this theorem is sometimes known as the parallelogram law. See also The Source of the Ring and The Field We are in line with the development of a set of factors in the theory of the Group. Definition If I is the ideal R ring and ∈ R then one of my cosets is a set of forms a + I = {a + s | s I }. Set all cosets signified by R/I. Remarks This is the same as a group theory coset (additive). As in the sets of case partitions the coset rings into a subset of disjoints. Element a is called coset coset representative. Note that each coset will have many cosmos representatives. The elements a1 and a2 represent my same coset if a1-a2 ∈ I. My own ideal is one coset: 0 + I. Teorem If I am the ideal R ring, the R/I set is the ring under operation (a + I) + (b + I) = (a + b) + me and (a + I) . (b + I) = (ab) + Me. Evidence We need to check that the operation is well defined. That is if a1 a2 are representatives of the same coset and b1 and b2 represent the same coset then a1 + b1 and a2 + + represents the same coset and so does a1b1 and a2b2. We have a1 - my a2 and b1 - my b2 etc adding this shows that (a1+ b1) – (a2+ b2) my ∈ and so this represents the same coset. Similarly, for the product, note that a1b1 - a2b2 = (a1 - a2)b1 + a2(b1 - b2) and the following results are of ideal properties. Once you know that the operation is well defined the axis of the ring follows easily. Note that zero ring factor is my own 0+ coset or my own ideal. Statement We call this ring as a ring of factors instead of ringing quota to avoid confusion with something called quota rings. The way you should think of the R/my ring factor is that it's an Ring Ring where all the elements in my ideal have been made into zero. This is sometimes called the murder of my elements. Here's a picture: Example Let me = nZ ⊆ Z. Then Z /nZ has elements n: Me, 1 + I, 2 + me, ... , (n - 1) + I and the factor ring is Zn. Let me become the ideal R[x] produced by polinomial x2 + 1. Then each coset has a representative form (a + bx) + me because if we choose a coset representative (say) x3 then we have x3 = -x + (x2 + 1)x - x + me and so on -x will do for the representatives instead. You should see that by thinking of x2 + 1 as 0 one can use the x2 = -1 rule to eliminate all higher power. So we get a ring where we can add and breed things like + bx using x2 = -1 rules to do ratings. If you replace x by i that is unexpected, you will see that this is a complex number. Namely: R[x]/ < x2+ 1 > C . Using the same argument we can see that the incense field of GF(9) built earlier is actually the factor of the Z3 ring[x]/< x2 + 1 > since the rating rule x2 = -1 we used is the equivalent of x2 + 1 = 0. The Z5 ring[x]/< x2 + 1 > has 25 elements since (as above) coset representatives can be selected to be a form a+ bx with a, b ∈ 5. However, this is not a field because coset (x + 3) + me and (x + 2) + I have a product (x + 3)(x + 2) + I = 0 + I. Therefore the ring has zero-divisor instead of fields. Note that the reason that this happens is that polinomial x2 + 1 factor in Z5[x] becomes a product of a lower degree factor. (You can see it has a linear factor because polinomials have roots in 5.) We looked at the last part that the homomorphism content of the ring was ideal and the image was subgropage. Given the corresponding results for the group it would not be surprising that we have Isomorphism Theorem for Rings If f :R→ S is homomorphism ringing factor R/ker(f) isomorphic to im(f). Proof of Specify θ map: R/ker(f)→ im(f) by + ker(f)) = f(a). Then the evidence is exactly as in the case of group theory unless you should also check that this map respects the multiplication of the ring as well as the addition. This statement indicates that one can always think of a factor ring as a homomorphic image rather than as something involving subset operations (cosets). From this theorem each ideal is the reirection of the some homomorphism (map from R to R/I). The above results are sometimes called the First Isomorphism Theorem for the Rings. Here are a few others. The Second Isomorphism theorem for my Let's Ring and J became the ideal R. Then I + J and I ∩ J are also suitable and the factor rings (I + J)/J and I / (I ∩ J) are isomorphic. Proof Here is a picture showing entry. Double lines represent two factor rings. To prove the result, Determine θ : → (I + J)/J with i ↦ i + J. Because each incoming coset (I+ J)/J has a representative of the θ form is to. If I ∈ ker(θ) then i + J = J and so i ∈ J and therefore I ⊆ J. The result is now following from the First Isomorphism Theorem. The truth is that the only one that I need to be subring R for this theorem to hold. One needs J to be ideal. Example Let Me = & lt; m > and J = & n > become ideal Z. Then (See Exercise 4 Qu 1) I + J = &amp lt; gcd(m, n) > and I ∩ J = < lcm(m, n) > . Then from this theorem one concludes that & lt; gcd(m, n) > /lt; n > & n > / < lcm(m, n) > and from this you can conclude that n/gcd(m, n) = lcm(m, n)/m or gcd(m, n) × lcm(m, n) = mn (Exercise 3) Third Isomorphism theorem for my Let Ring and J became the ideal R ring with ⊆ J. Then J/I was ideal R/I and (R/I)/(J/I) R/J Easy Proof to confirm that J/I is the ideal R/I. Specify map θ : R/I→ R/J by + I ↦ a + J for any coset a+ I in R. This is clearly inside. Element a + I am in imil θ if a + I = J which (because I ⊆ J) will only happen if ∈ J. H. Hence the ker(θ) = J/⊆ R/I and the result is followed by the First Isomorphism Theorem. JOC/EFR 2004 The statement was the First Isomorphism Theorem for the Ring of the Algebra Abstract by Dummit and Foote. I want to check whether it's all ok. In particular I was a bit worried about (*) lines. It looks a bit awkward, but is it too bad? Any suggestions? Theorem: 1) If $\varphi:R \ rightarrow S$ is a homomorphism ring, then the isirel φ is ideal $$R$, the image $\varphi $ is subring of the S and $R/ker \varphi $isomorphic as the ring to $\varphi $varphi(R)$. 2) If I is any ideal $$R, then the $R map \rightarrow R/I$ is defined by $r \rightarrow r+I$ is a surcective ring homomorphism with a I. Therefore, each ideal is the homomorphism kernel of the ring and vice versa. Proof: Leave $\varphi: R \rightarrow S$ becomes ring homomorphism. If $r \in R$ and $r' \in the ker \varphi$, then we have a $rr',r'r\in ker \varphi$ (so that it is closed under multiplication by the elements of R) since $\varphi (rr')=\varphi (r).\varphi (r')=\varphi (r)0 =0=0 \varphi (r)=\varphi (r') \varphi (r)=\varphi (r'r)$$ since $ker \varphi$ is also a subring of R, it is ideal to R. It's clear $\varphi (R)$ is a subring S. Now, $I $ becomes ideal $$R, so that R/I is also a ring, and defines $\pi:R \rightarrow R/I$ by $\pi (r)=r+I$. We know π is a group homomorphism with kernel I, and for $r, s \in R$, we have $$\pi (rs)=(rs)+I=(r+I)(s+I)=\pi (r) \pi (s), $ $ so $\pi $ actually homomorphism. Tentukan kemudian $\phi: R/ker \varphi \rightarrow \varphi(R)$ oleh $$\phi (r+(ker \phi))= \varphi (r),$$ untuk setiap $(r+(ker \phi)) \dalam R/ker \varphi$, untuk sesetengah $r \dalam R$. This is well defined because if $r' \in (r+(ker \varphi)),$ then $$\phi (r'+(ker \varphi))=\varphi(r')=\varphi(r)=\phi (r+(ker \varphi)).$$ Also, this is a ring isomorphism because for each $\varphi (s) \in \varphi (R)$ for some $s \in R$, we have $$(*) \ \phi^{-1}\{\varphi (s)\}= \phi^{-1} \varphi[r+(ker \varphi)]= \phi^{-1} \varphi[\pi^{-1} \{r+(ker \varphi)\}]= \{r+(ker \varphi)\},$$ a set with a single element of $R/ker \varphi$, so that it is a bijection, and for every $r+(ker \varphi), r'+(ker \varphi) \in R/ker \varphi$, for some $r,r' \in R$, we have $$\phi [(r+(ker \varphi))+(r'+(ker \varphi))]=\phi [(r+r')+(ker \varphi)]=\varphi(r+r')=\varphi(r)+ \varphi(r')=\phi [r+(ker \varphi)]+ \phi [r'+(ker \varphi)] $$1 dan $\phi [(r+(ker \varphi)) (r'+(ker \varphi)]=\phi [(rr')+(ker \varphi)]=\varphi(rr'))=\varphi(rr')=\varphi(r) \varphi(r')=\phi [r+(ker \varphi)] \phi [r'+(ker \varphi)] ,$$ supaya ia adalah homomorphism cincin. homomorphism.

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